Numerical Modeling of a Proton Beam in a Transport Line from a Cyclotron to an Experimental Target

Bachelor Thesis Defence
of

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2025

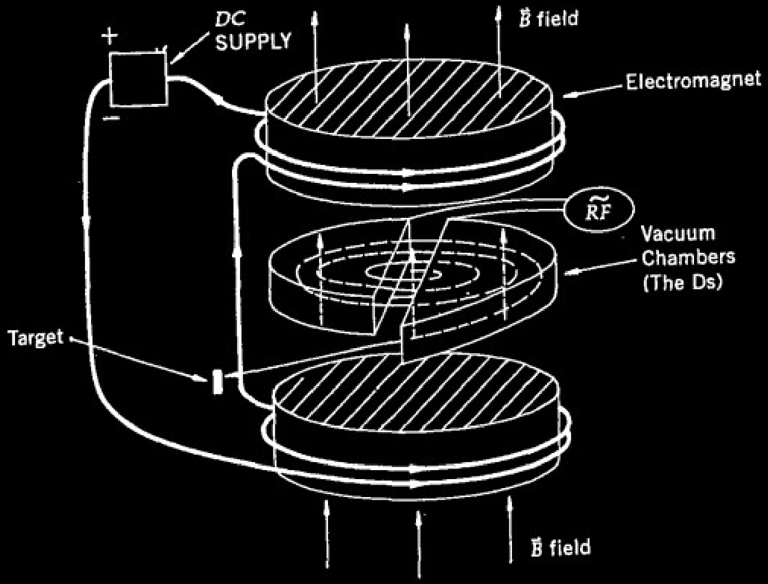
Overview

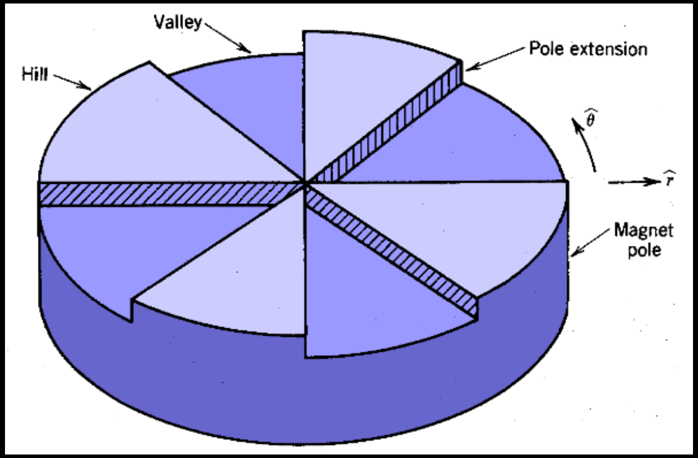
We will simulate the beam dynamics of two transport lines with the MAD-X program and compare our results with the ones already obtained using TraceWin.

- Cyclotrons as charged particle accelerators
- Types of beam focusing
- The Matrix method and the MAD-X program
- The setup
- Results of our simulations

Cyclotrons as charged particle accelerators

- a particle source placed between two vacuum chambers in constant magnetic field(decreases with radius -> non-relativistic energies)
  - modern cyclotrons – a sequence of magnets and cavities – Azimuthally Varying Focusing(AVF) cyclotrons, relativistic energies, magnetic field increases with the radius



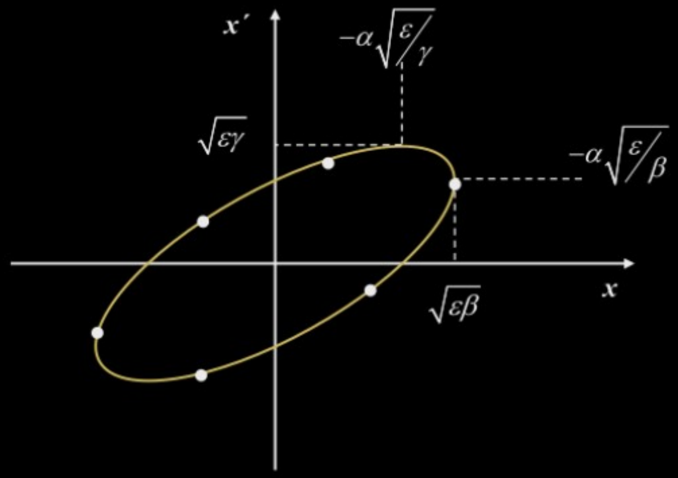


Weak focusing

- rely only on weak focusing
- field varies with radius – radial field lines – vertical forces – betatron oscillations(in the transverse planes)
- divergence angle and emittance

$$x' = \frac{dx}{ds} \qquad \text{Area} = \pi\varepsilon \ \ [\text{mm} \cdot \text{rad}]$$

- weak due to the not large gradients applied

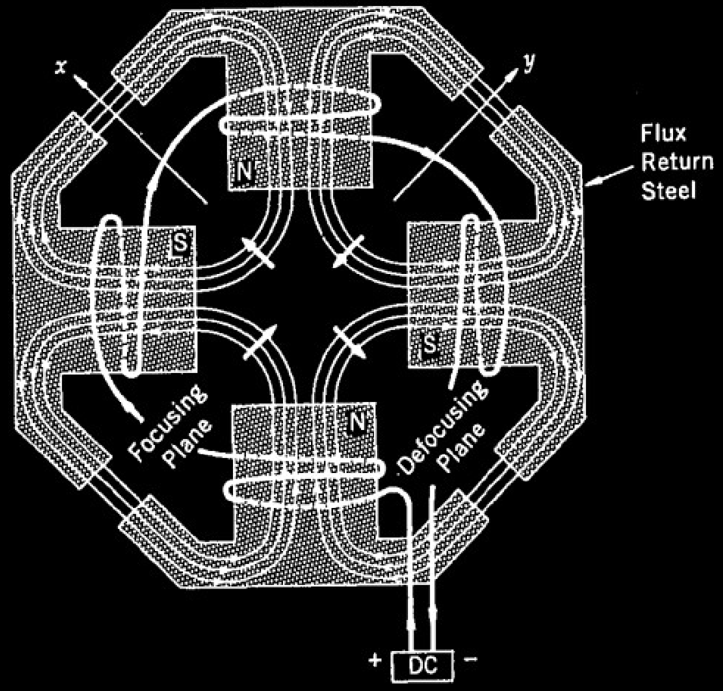


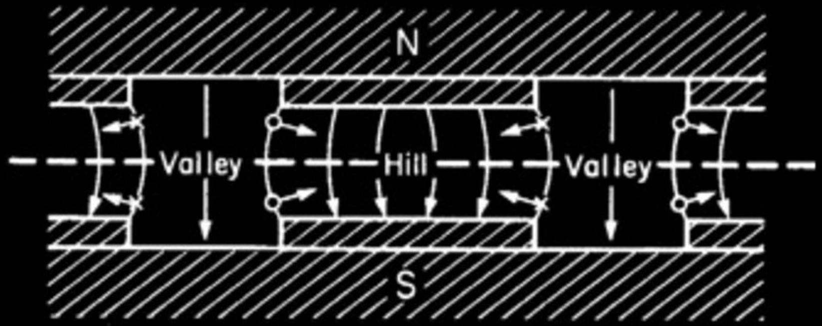
Strong focusing

- additional quadrupole magnets
- focus in one plane, defocus in the other
- placed in a sequence – the lattice
- quadrupole strength: angular deflection:

$$k = \frac{1}{(B\rho)} * \frac{dB_y}{dx} \qquad \Delta x' = \theta = lkx$$

- equation of motion: $y'' + k(s)y = 0$





The Matrix method

- equation of motion - > second order linear differential equation
- transport matrix – trace the solutions

$$y = \sqrt{\epsilon\beta(s)}\sin(\mu(s) + \mu_o)$$

$$\begin{bmatrix} y(s_2) \\ y'(s_2) \end{bmatrix} = M_{21} \begin{bmatrix} y(s_1) \\ y'(s_1) \end{bmatrix}$$

- general transport matrix for one revolution

$$M = \begin{bmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{bmatrix}$$

here μ – phase advance

- symplectic matrix - > implies preservation of phase space volume

$$area = \int pdq = const$$

$$M^T J M = J, where J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$$

Courant-Snyder parameters

$$\epsilon = \gamma y^2 + 2\alpha y y' + \beta y'^2$$

envelopes

$$E(s) = +/-\sqrt{\epsilon\beta(s)}$$

dispersion function

$$x(s) = D(s)\frac{\Delta p}{p_o}$$

The MAD-X program

- Methodical Accelerator Design
- optics and lattice calculations based on machine characteristics
- developed at CERN, free distribution, works on all platforms
- optical functions, particle tracking, optimization(matching)…
- single-particle tracking

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  +     MAD-X 5.09.01  (64 bit, Linux)       +
  + Support: mad@cern.ch, http://cern.ch/mad +
  + Release   date: 2023.12.04               +
  + Execution date: 2025.09.09 00:50:05      +
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X:>
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QF: QUADRUPOLE, L=0.24989 , K1:=kqf;
QD: QUADRUPOLE, L=0.24989, K1:=kqd;

QFM: QF, K1:=kqfm;
QDM: QD, K1:=kqdm;

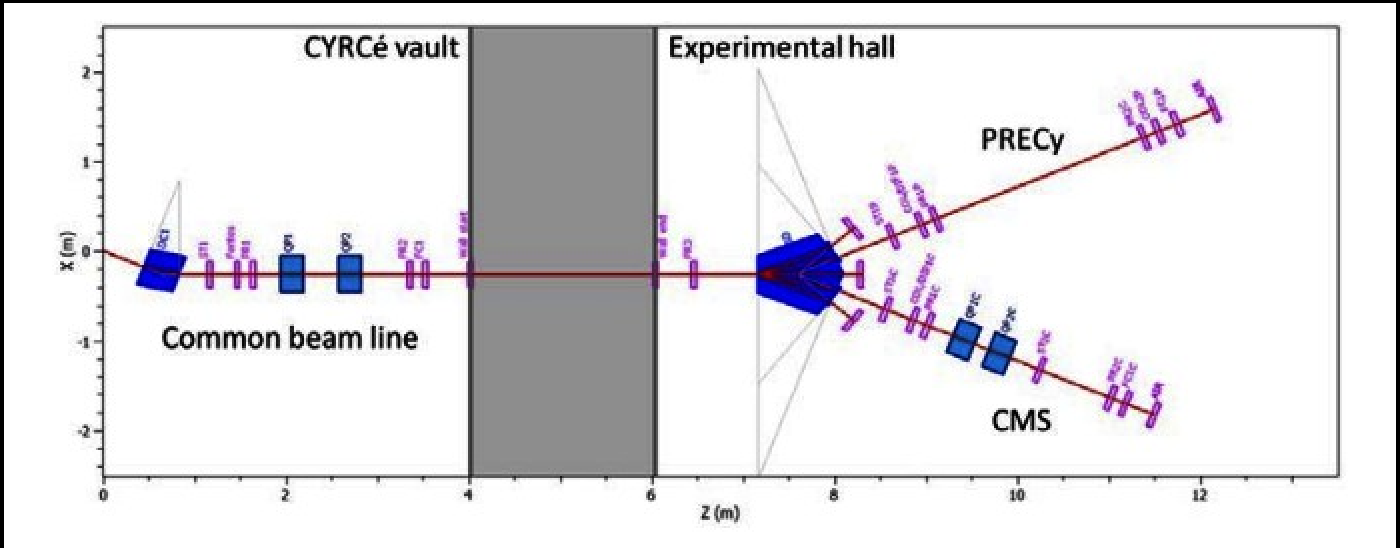
BM: RBEND, ANGLE=-0.384, HGAP= 0.025, L=0.4;
BE: SBEND, ANGLE=0.384, L=0.93715, HGAP=0.03;
BEL: SBEND, ANGLE=-0.384, L=0.93715, HGAP=0.03;

BEAM, PARTICLE=PROTON, ENERGY=0.9633, EX=0.0000019, EY=0.0000037;

mainline: SEQUENCE, L=11.838;
bem: BM, AT=0.645;
qdc: QFM, AT=2.099;
qfc: QDM, AT=2.750;
```

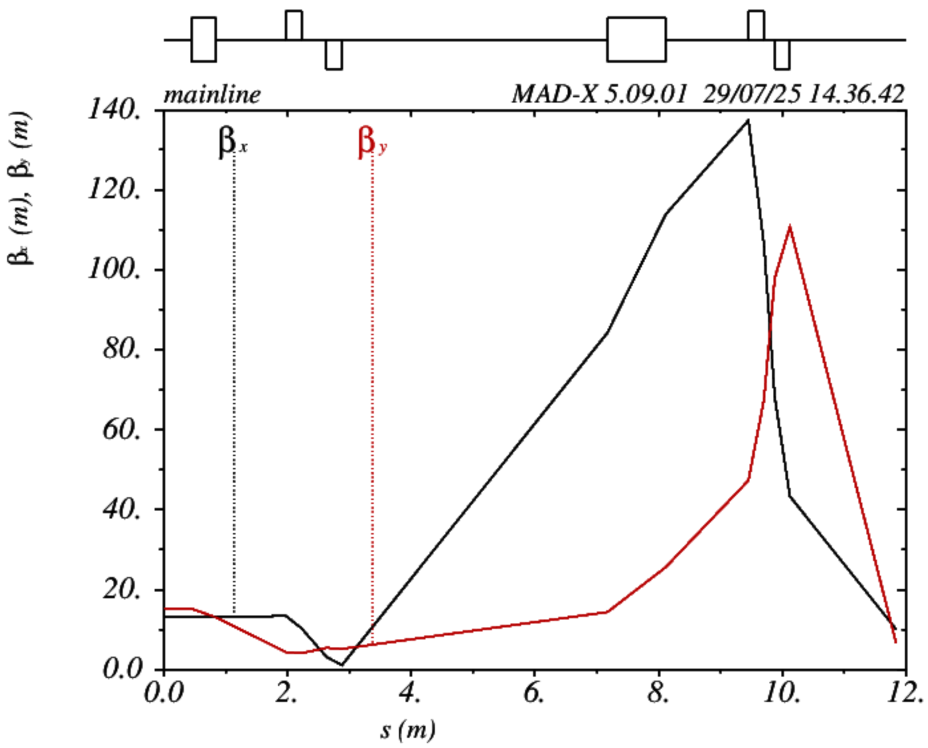
The CYRCé setup

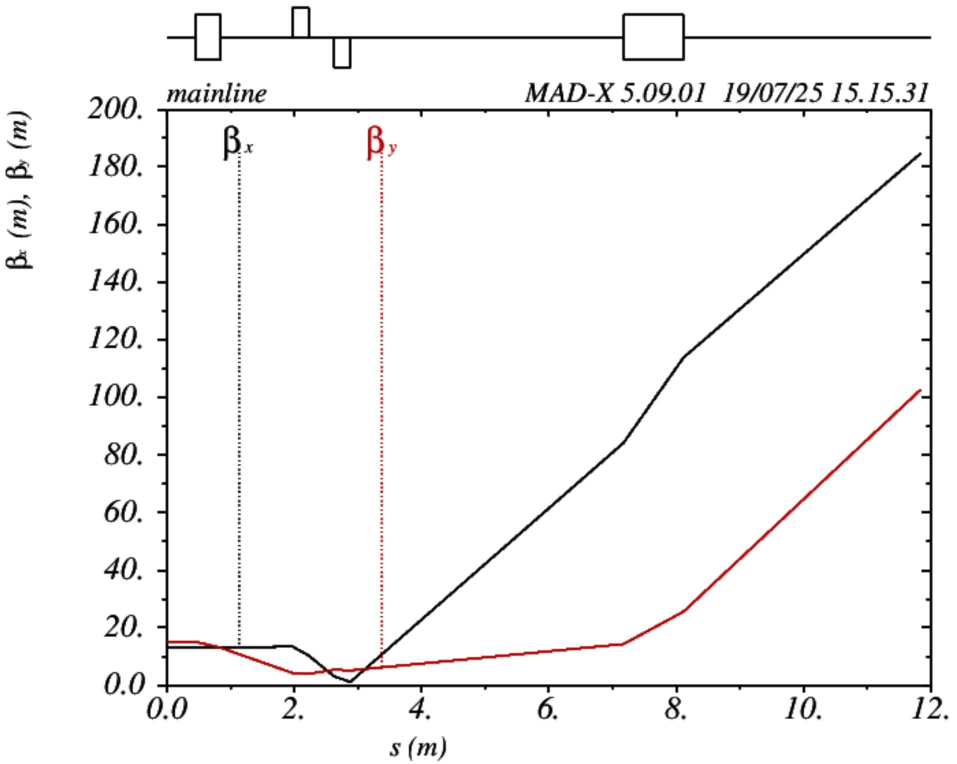
- located in Strasbourg, Institut Pluridisciplinaire Hubert Curien (IPHC)
- TR-24 cyclotron, for the production of radioisotopes
- two experimental stations – CMS silicon modules and PRECy(Platform for Radiobiological Experiments at CYRCé)



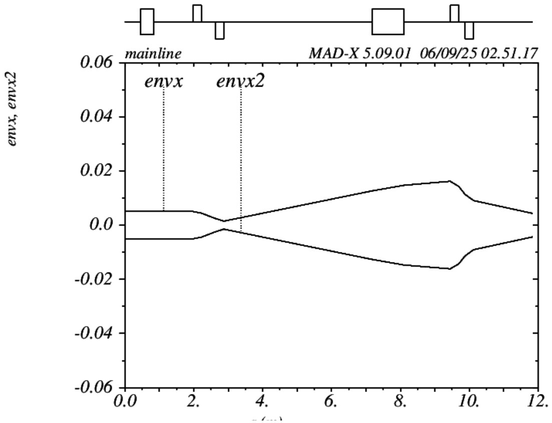
Plotting the amplitude functions

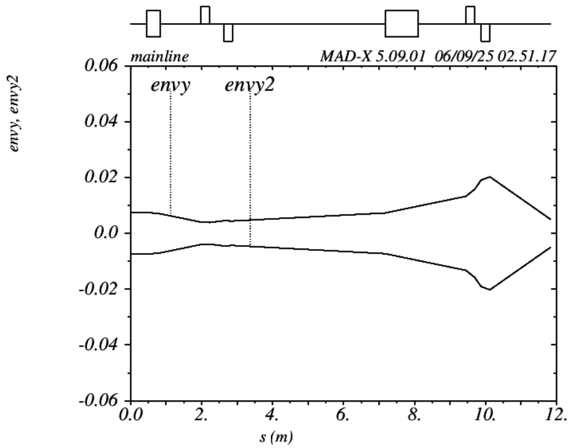
- typical behavior of the functions in a FODO cell



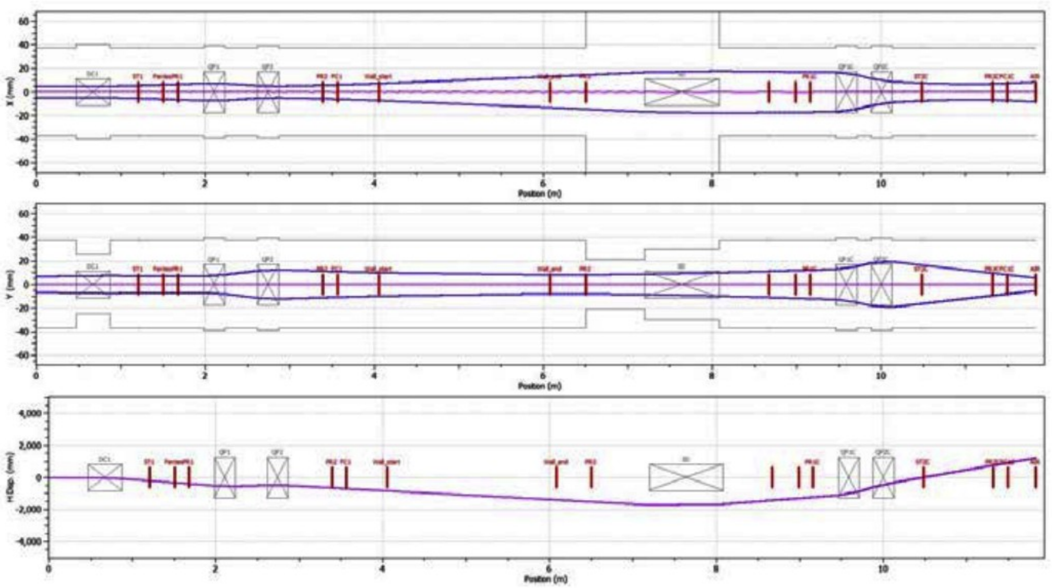


The beam envelopes – CMS line

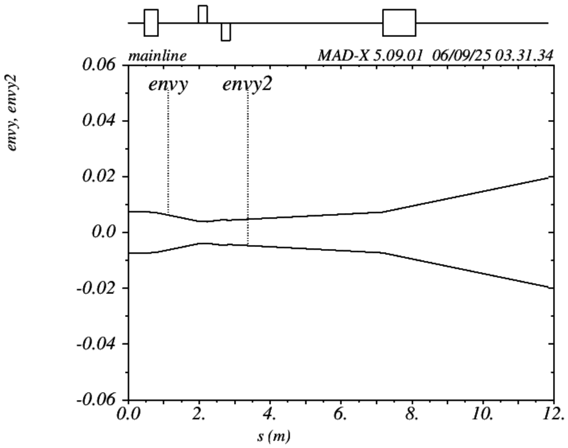




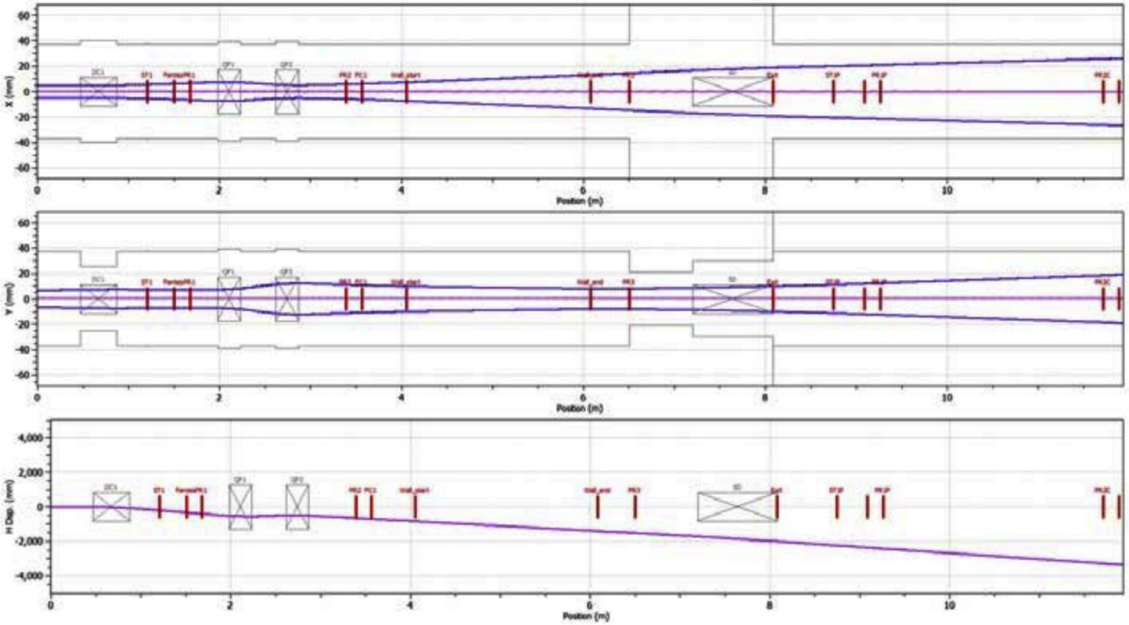
- beam shape – very similar and 1.8 cm in the x direction and 2.0 cm in the y-direction.

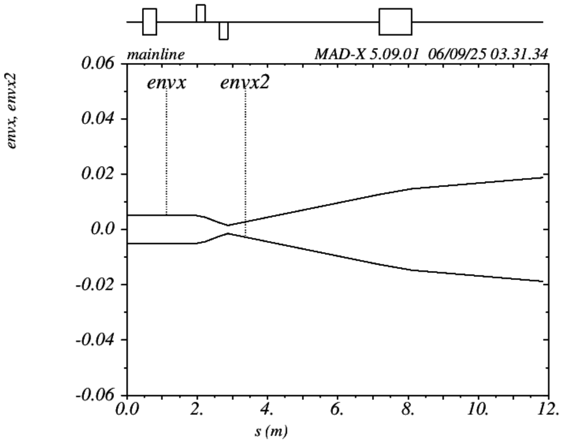


The beam envelopes - PRECy line



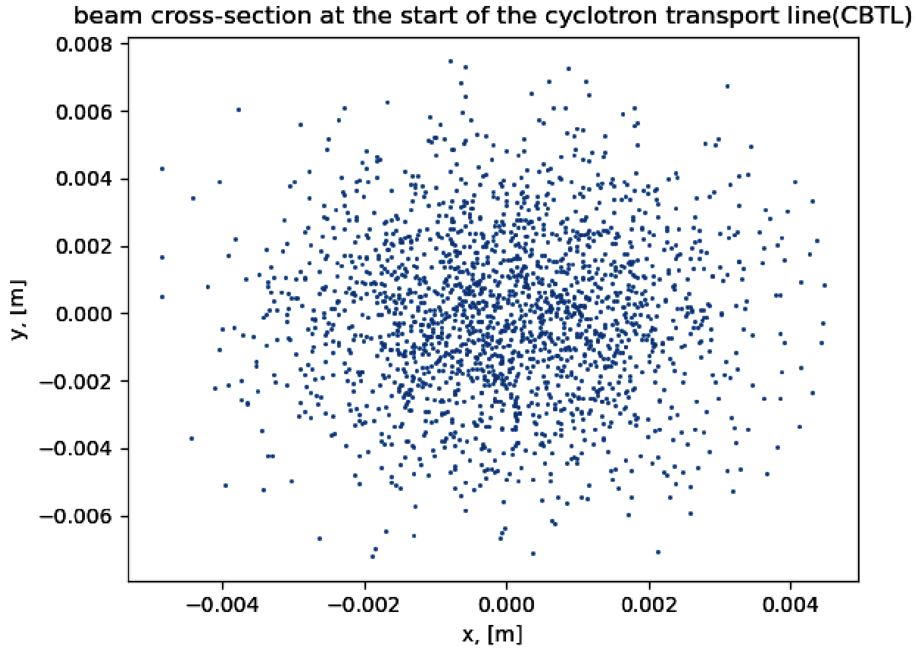
similar shape, but x = 1.9 cm =/= 2.6 cm in TraceWin – with dp/p=0

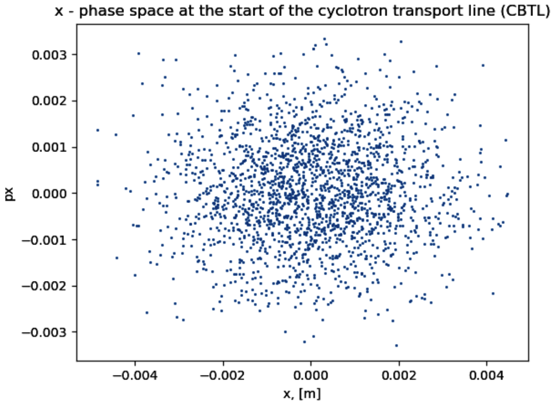


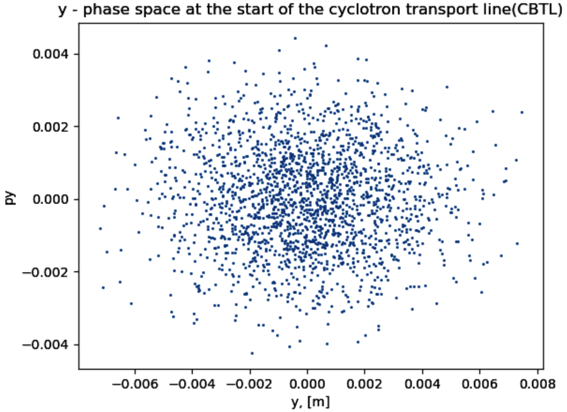


Particle tracking – start of the common line

- generate 2000 particle coordinates in phase space, assumption – rectified ellipse

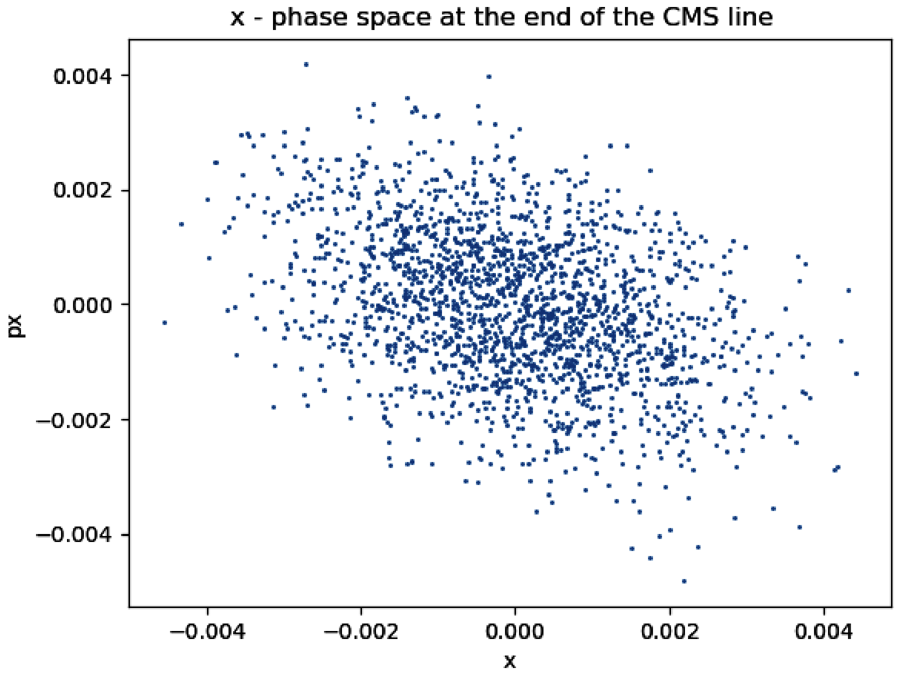


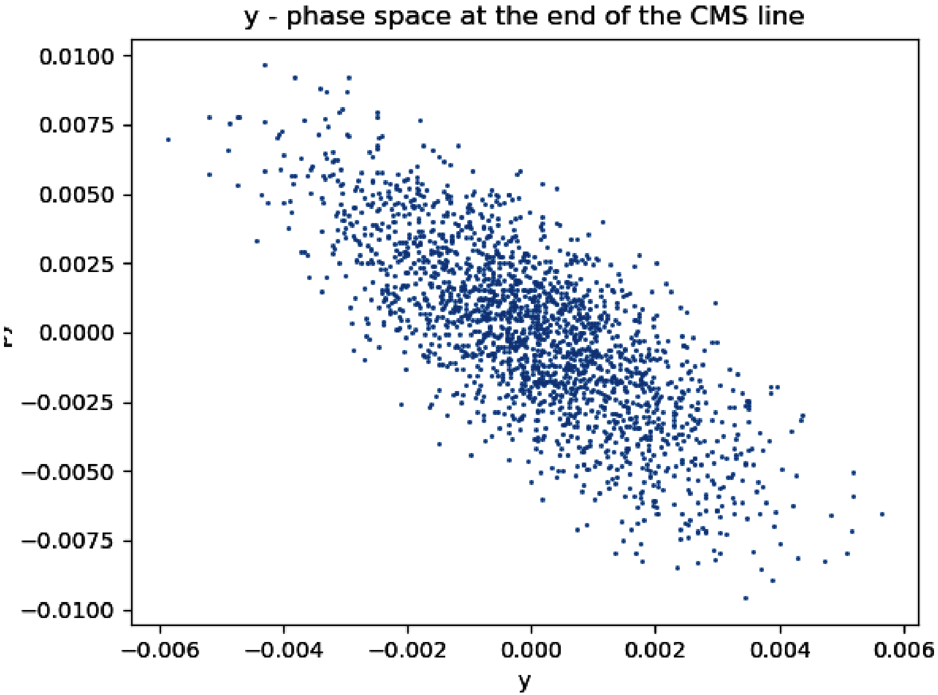




PS distribution of the CMS line

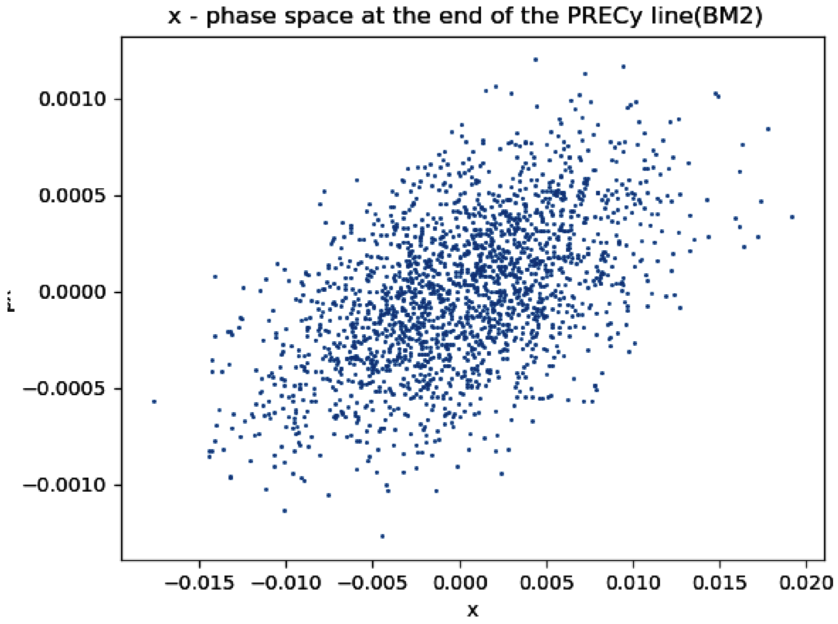
- RMS_x = 1.48 mm RMS_y = 1.69 mm MAD-X
- RMS_x = 2.8 mm RMS_y = 1.8 mm TraceWin

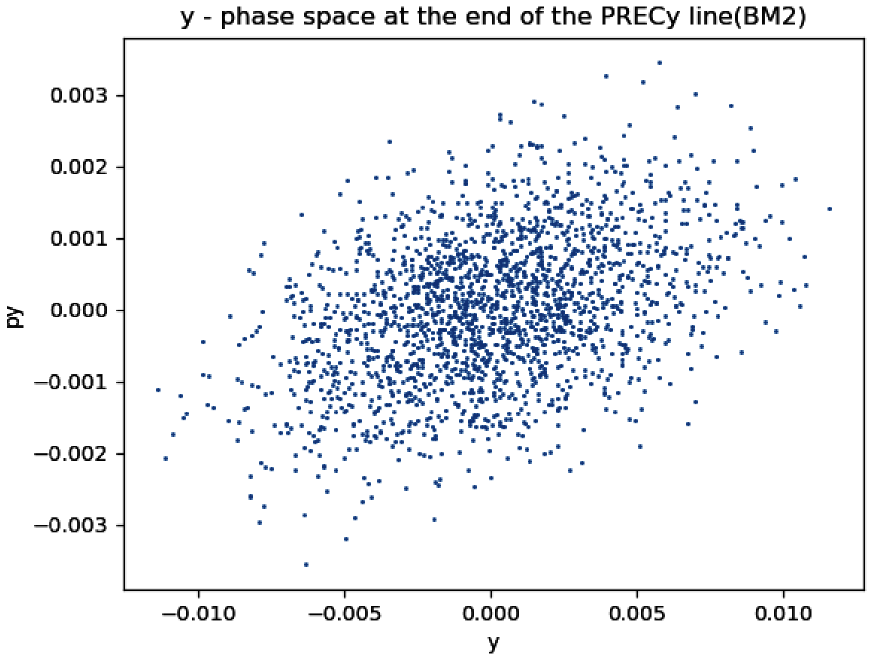




PS distribution of the PRECy line

- FWHM$_x$ = 12.95 mm   FWHM$_y$ = 8.88 mm   MAD-X
- FWHM$_x$ = 13 mm   FWHM$_y$ = 8 mm   TraceWin





Conclusion

- We obtained similar results with the MAD-X program, in comparison to the ones obtained with TraceWin, but not identical
- beamline elements have been neglected, zero dispersion assumed and a particular initial distribution in phase space

Thank you for your attention!

References

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