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Thesis defence for obtaining the educational qualification degree of Bachelor

Aerodynamic Characteristics of the Aircraft Wing

Bachelor of Science in Nuclear and Particle Physics

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Outline

Aim of the Thesis: Investigate the aerodynamic flow around airfoils through the application of the Joukowski transformation: a conformal mapping technique that converts a circle into a symmetric or cambered airfoil shape.

1. Fundamental equations of fluid mechanics
2. Axisymmetric airflow around a circle
3. Joukowski airfoil theory
4. Airflow models around Boeing 737-800 wing

Basic Principles of Fluid Mechanics

The fluid motion is determined by three fundamental laws of physics:

1. **Conservation of mass**, expressed by the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0.$$

2. **Conservation of linear momentum** (Navier-Stokes equations), based on Newton's second law of motion:

$$\rho \left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = -\nabla p + \nabla \cdot \hat{\tau} + \rho \vec{f}.$$

3. **Conservation of energy**(first law of thermodynamics):

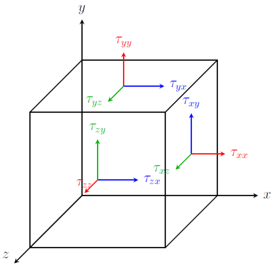
$$\rho \left(\frac{\partial \varepsilon}{\partial t} + \vec{u} \cdot \nabla \varepsilon \right) = -p(\nabla \cdot \vec{u}) + \Phi + \nabla \cdot (\kappa \nabla T) + \rho \dot{q}.$$

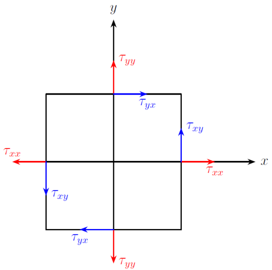
The Viscous Stress Tensor

The viscous stress tensor is given by:

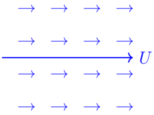
$$\hat{\tau} = \underbrace{\mu\left(\nabla\vec{u} + (\nabla\vec{u})^T\right)}_{\text{Rate-of-strain tensor } \hat{S}} - \underbrace{\frac{2}{3}\hat{1}\mu\nabla\cdot\vec{u}}_{\text{Shear deformations}} + \underbrace{\zeta\hat{1}\nabla\cdot\vec{u}}_{\text{Bulk viscosity}} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \tau_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{zy} & \tau_{zz} \end{pmatrix},$$

where $\vec{u}$ is the flow velocity, μ is the dynamic viscosity, ζ is the bulk viscosity.

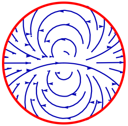




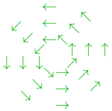
Axisymmetric Airflow Decomposition Around a Circle



Uniform Flow U



Doublet Flow (Cylinder)



Vortex Flow (Γ)

- Uniform flow (velocity U in x-direction):

$$\phi_{\text{uniform}} = Ux = Ur\cos\theta, \quad \psi_{\text{uniform}} = Uy = Ur\sin\theta.$$

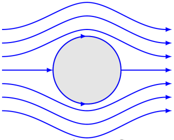
- Doublet flow (represents the cylinder: no flow penetration at $r = R$):

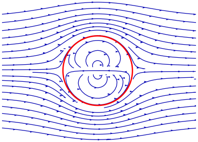
$$\phi_{\text{doublet}} = \frac{UR^2\cos\theta}{r}, \quad \psi_{\text{doublet}} = -\frac{UR^2\sin\theta}{r}.$$

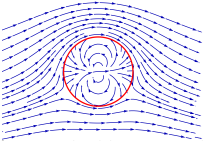
- Point vortex (adds circulation Γ):

$$\phi_{\text{vortex}} = \frac{\Gamma\theta}{2\pi}, \quad \psi_{\text{vortex}} = -\frac{\Gamma}{2\pi}\ln\frac{r}{R}.$$

Combined flow


Uniform flow


Uniform + doublet flow


Uniform + doublet + circulation

- Combined potential and stream function:

$$\phi = \phi_{\text{uniform}} + \phi_{\text{doublet}} + \phi_{\text{vortex}} = U\left(r + \frac{R^2}{r}\right)\cos\theta + \frac{\Gamma\theta}{2\pi},$$

$$\psi = \psi_{\text{uniform}} + \psi_{\text{doublet}} + \psi_{\text{vortex}} = U\left(r - \frac{R^2}{r}\right)\sin\theta - \frac{\Gamma}{2\pi}\ln\frac{r}{R}.$$

- The velocity field in polar coordinates:

$$u_r = \frac{\partial\phi}{\partial r} = U\left(1 - \frac{R^2}{r^2}\right)\cos\theta, \quad u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -U\left(1 + \frac{R^2}{r^2}\right)\sin\theta + \frac{\Gamma}{2\pi r}.$$

- The pressure distribution p using Bernoulli's equation:

$$p = p_\infty + \frac{1}{2}\rho U^2 - \frac{1}{2}\rho|\vec{u}|^2, \quad |\vec{u}|^2 = u^2 + v^2, \quad U = \lim_{r\to\infty} u(r,\theta).$$

- Lift and drag force:

$$L = \rho U\Gamma \text{ (Kutta-Joukowski)}, \quad D = 0 \text{ (D'Alembert's paradox)}.$$

Joukowski Airfoil Theory of a Circle in 2d

- Complex potential:

$$\Phi(z) = \phi(z) + i\psi(z) = U\left[(z - z_0) + \frac{R^2}{z - z_0}\right] \pm \frac{i\Gamma}{2\pi}\ln\frac{z - z_0}{R},$$

- Complex velocity:

$$W = \frac{\partial\Phi}{\partial z} = u - iv,$$

- Complex number:

$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta},$$

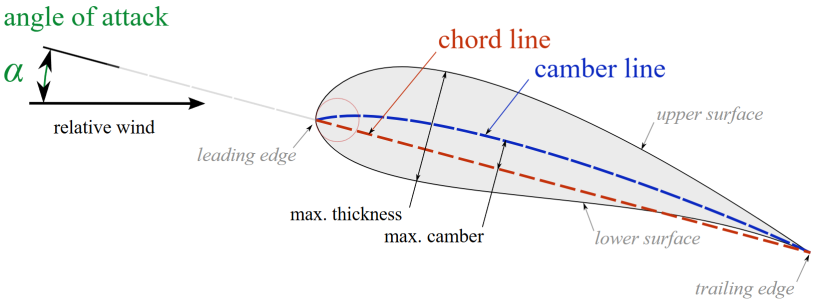
- The Blasius theorem:

$$F_x - iF_y = \frac{i\rho}{2}\oint\left(\frac{d\Phi}{dz}\right)^2 dz, \quad \text{Drag: } (F_x), \quad \text{Lift: } (F_y).$$

- Kutta-Joukowski Lift and Drag for an ideal fluid:

$$F_x = 0 \quad (\text{Drag}), \quad F_y \equiv L = \pm\,\rho U\Gamma \quad (\text{Lift}).$$

Basic Characteristics of the Airfoil



Joukowski Transformation

The transformation maps the z-plane (circle) to the ζ-plane (airfoil):

$$\zeta(z) = z + \frac{c^2}{z}.$$

To create an airfoil with a sharp trailing edge, the circle in the z-plane must pass through the point $z = c \in \mathbb{R}^+$. For a circle with center $z_0 = m + in$, where $m, n \in \mathbb{R}$, the radius R is determined by:

$$R = |c - z_0| = \sqrt{(c - m)^2 + n^2}.$$

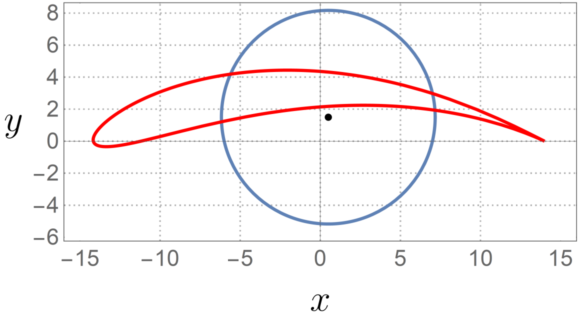
The inverse transformation is:

$$z(\zeta) = \frac{\zeta + \sqrt{\zeta^2 - 4c^2}}{2}.$$

The Joukowski transformation uses the parameters of the initial circle to determine the final shape of the airfoil. Here is how each parameter affects the geometry.

An Example of Joukowski Transformation

Joukowski transformation of a circle with radius $R = 6.67$ and parameters: $z_0 = (0.5, 1.5) = (m, n)$, $c = 7$:



Summary

- We derived the fundamental equations of fluid mechanics – expressed in both integral and differential forms – based on the conservation of mass, momentum, and energy, specifically for steady, incompressible flows with constant density and in the absence of external forces.

- We studied a two-dimensional, steady, uniform flow past a circular cylinder of radius R in the Kutta–Joukowski setup, deriving key flow quantities such as the velocity potential, stream function, pressure distribution, and the lift and drag forces.

- We considered the two-dimensional Joukowski airfoil theory using complex variables. By applying the Joukowski transformation to map a circle into an airfoil, we enable an analytical study of ideal, inviscid flow around the airfoil.

- Finally, we analyzed a specific real-world airfoil-shaped wing by examining simulated velocity and pressure distributions around a Boeing 737-800 wing, illustrating the practical relevance of the theory and supporting the earlier theoretical results on lift generation.

Thank You!